Module V

Truth values and Tables in Fuzzy Logic, Fuzzy propositions, Formation of fuzzyrules - Decomposition of rules – Aggregation of rules, Fuzzy Inference Systems –Mamdani and Sugeno types, Neuro-fuzzy hybrid systems – characteristics –classification.

Truth values and Tables in Fuzzy Logic

Fuzzy logic uses linguistic variables. The values of a linguistic variable are words or sentences in a natural orartificial language.

For example, height is a linguistic variable if it rakes values such as tall, medium, shortand so on. The linguistic variable provides approximate characterization of a complex problem.

The name ofthe variable, the universe of discourse and a fuzzy subset of universe of discourse characterize a fuzzy variable.A linguistic variable is a variable of a higher order than a fuzzy variable and its values are taken to be fuzzy variables.

A linguistic variable is characterized by:

1.name of the variable (x);
2. term set of the variable *t(x);*
3. syntactic rule for generating the values of *x;*
4. semantic rule for associating each value of *x* with its meaning.

Apart from the linguistic variables, there exists what are called as *linguistic hedges (linguistic modifiers).*

Examples of linguistic hedges

In the fuzzy set "very tall", the word "very" is a linguistic hedge. A few popular linguistic hedgesinclude: very, highly, slightly, moderately, plus, minus, fairly, rather.

Reasoning has logic as its basis, whereas propositions are text sentences expressed in any language and aregenerally expressed in an canonical form as

zisP

where**z**→ the symbol of the subject and

P→ the predicate designing the characteristics of the subject.

Example

"**London is in United Kingdom**"
"London" → the subject and

"in United Kingdom" →the predicate, which specifies a property of"London," i.e., its geographical location in UnitedKingdom.

Negation

Every proposition has its opposite, called **negation**.

Logic Functions

- Truth cables define logic functions of two propositions.
- Let *X* and *Y* betwo propositions, either of which can be true or false.
- The basic logic operations performed over the propositions are the following:

I. Conjunction (Λ) :X AND Y.
2. Disjunction (∨) :XOR Y.
3. Implication or conditional (=>): IF X THEN Y.
4. Bidirectional or equivalence (⇔): X IF AND ONLY IF Y.

Inference Rules

On the basis of these operations on propositions, inference rules can be formulated. Few inference rulesare as follows:

$$[X \wedge (X \Rightarrow Y)] \Rightarrow Y$$
$$[\bar{Y} \wedge (X \Rightarrow Y)] \Rightarrow \bar{X}$$
$$[(X \Rightarrow Y) \wedge (Y \Rightarrow Z)] \Rightarrow (X \Rightarrow Z)$$

The above rules produce certain propositions that are always true irrespective of the truth values ofpropositions *X* and *Y.* Such propositions are called **tautologies**.

An extension of set-theoretic bivalence logic isthe fuzzy logic where the truth values are terms of the linguistic variable "truth."

The truth values of propositions in fuzzy logic are allowed to range over the unit interval [0, 1]. A truthvalue in fuzzy logic "very true" may be interpreted as a fuzzy set in [0, I]. The truth value of the proposition" *Z* is *A,"* or simply the truth value of *A,* denoted by tv(A) is defined by a point in [0, 1] (called the numerical truth value) or a fuzzy set in [0, 1] (called the linguistic truth value).

The truth value of a proposition can be obtained from the logic operations of other propositions whose truth values are known. If tv(X) and tv(Y) are numerical truth values of propositions *X* and *Y,* respectively, then

$$tv(X \text{ AND } Y) = tv(X) \wedge tv(Y) = \min\{tv(X), tv(Y)\} \quad \text{(Intersection)}$$

$$tv(X \text{ OR } Y) = tv(X) \vee tv(Y) = \max\{tv(X), tv(Y)\} \quad \text{(Union)}$$

$$tv(\text{NOT } X) = 1 - tv(X) \quad \text{(Complement)}$$

$$tv(X \Rightarrow Y) = tv(X) \Rightarrow tv(B) = \max\{1 - tv(X), \min[tv(X), tv(Y)]\}$$

FUZZY PROPOSITIONS

The fuzzy propositions are as follows:

1. **Fuzzy predicates:**
   In fuzzy logic the predicates can be fuzzy, for example, tall, short, quick. Hence, we haveproposition like "**Peter is tall**."

2. **Fuzzy-predicate modifiers:**
   In fuzzy logic, there exists a wide range of predicate modifiers that act as hedges.
   For example, very, fairly, moderately, rather, slightly. These predicate modifiers are necessary for generatingthe values of a linguistic variable. An example can be the proposition "**Climate is moderately cool**," where"moderately" is the fuzzy predicate modifier.

3. **Fuzzy quantifiers:**
   The fuzzy quantifiers such as most, several, many, frequently are used in fuzzy logic.Employing these, we can have proposition like "**Many people are educated**."
   A fuzzy quantifier can beinterpreted as a fuzzy number or a fuzzy proposition, which provides an imprecise characterization of thecardinality of one or more fuzzy or non fuzzy sets. Fuzzy quantifiers can be used to represent the meaning of propositions containing probabilities; as a result, they can be used to manipulate probabilities withinfuzzy logic.

4. **Fuzzy qualifiers:**

   There are four modes of qualification in fuzzy logic, which are as follows:

   (i) **Fuzzy truth qualification:** It is expressed as "**x is τ** ," in which **τ** is a fuzzy truth value. A fuzzy truthvalue claims the degree of truth of a fuzzy proposition.

Example

(Paul is Young) isNOT VERY True.

Here the qualified proposition is (Paul *is* Young) and the qualifying fuzzy truth value is "NOT Very True."

(ii)Fuzzy probability qualification:

It is denoted as **"x is λ"**where, **λ**is fuzzy probability.

In conventionallogic, probability is either numerical or an interval.

In fuzzy logic, fuzzy probability is expressed byterms such as likely, very likely, unlikely, around and so on.

Example

(Paul is Young) is Likely.

Here the qualifying fuzzy probability is "Likely." These probabilities may be interpreted as fuzzynumbers, which may be manipulated using fuzzy arithmetic.

(iii) Fuzzy possibility qualification:

It is expressed as **"x is π",**where **π**is a fuzzy possibility and can be of thefollowing forms: possible, quite possible, almost impossible. These values can be interpreted as labelsof fuzzy subsets of the real line.

Example

(Paul is Young) is Almost Impossible.

Here the qualifying fuzzy possibility is "Almost Impossible."

(iv) Fuzzy usuality qualification:

It is expressed as **"usually (X) =usually (X is F),"**in which the subject Xis a variable taking values in a universe of discourse U and the predicate F is a fuzzy subset of U andinterpreted as a usual value of X denoted by U(X) = F. The propositions that are usually true or theevents that have high probability of occurrence are related by the concept of usuality qualification.

FORMATION OF FUZZY RULES

The general way of representing human knowledge is by forming natural language expressions given by

IF antecedant THEN consequent.

The above expression is referred to as the IF - THEN rule based form. There are three general forms that existfor any linguistic variable. They are:

(a) Assignment statements;
(b) Conditional statements;
(c) Unconditional statements.

1. **Assignment statements:**

They are of the form

y =small
Orange color = orange
a=s
Paul is not tall and not very short
Climate = autumn
Outside temperature = normal

These statements utilize "=" for assignment.

2. **Conditional statements:**

The following are some examples.

IF y is very cool THEN stop.
IF A is high THEN B is low ELSE B is not low.
IF temperature is high THEN climate is hot.

The conditional statements use the "IF - THEN" rule-based form.

3. **Unconditional statements:**

They can be of the form

Gotosum.
Stop.
Divide by a.
Turn the pressure low.

DECOMPOSITION OF RULES

A compound rule is a collection of many simple rules combined together. Any compound rule structure may be decomposed and reduced to a number of simple canonical rule forms. The rules are generally basedon natural language representations.

The following are the methods used for decomposition of compoundlinguistic rules into simple canonical rules.

1. **Multiple conjunctive antecedents**

IFxis $\underset{\sim}{A}_1, \underset{\sim}{A}_2, \ldots, \underset{\sim}{A}_n$ THEN y is B_m.

Assume a new fuzzy set $\underset{\sim}{A}_m$ is defined as

$$\underset{\sim}{A}_m = \underset{\sim}{A}_1 \cap \underset{\sim}{A}_2 \cap \cdots \cap \underset{\sim}{A}_n$$

and expressed by means of membership function

$$\mu_{\underset{\sim}{A}_m}(x) = \min[\mu_{\underset{\sim}{A}_1}(x), \mu_{\underset{\sim}{A}_2}(x), \ldots \mu_{\underset{\sim}{A}_n}(x)].$$

In view of the fuzzy intersection operation, the compound rule may be rewritten as

IF $\underset{\sim}{A}_m$ THEN $\underset{\sim}{B}_m$.

2. **Multiple disjunctive antecedents**

$$\text{IF } x \text{ is } \underset{\sim}{A}_1 \text{ OR } x \text{ is } \underset{\sim}{A}_2, \ldots \text{ OR } x \text{ is } \underset{\sim}{A}_n \text{ THEN } y \text{ is } \underset{\sim}{B}_m.$$

This can be written *as*

$$\text{IF } x \text{ is } \underset{\sim}{A}_n \text{ THEN } y \text{ is } \underset{\sim}{B}_m$$

where the fuzzy set $\underset{\sim}{A}_m$ is defined as

$$\underset{\sim}{A}_m = \underset{\sim}{A}_1 \cup \underset{\sim}{A}_2 \cup \underset{\sim}{A}_3 \cup \cdots \cup \underset{\sim}{A}_n$$

The membership function is given by

$$\mu_{\underset{\sim}{A}_m}(x) = \max[\mu_{\underset{\sim}{A}_1}(x), \mu_{\underset{\sim}{A}_2}(x), \ldots, \mu_{\underset{\sim}{A}_n}(x)]$$

which is based on fuzzy union operation.

3. **Conditional statements (with ELSE and UNLESS)**

Statements of the kind

IF $\underset{\sim}{A}_1$ THEN ($\underset{\sim}{B}_1$ ELSE $\underset{\sim}{B}_2$)

can be decomposed into two simple canonical rule forms, connected by "OR":

IF $\underset{\sim}{A}_1$ THEN $\underset{\sim}{B}_1$
OR
IF NOT $\underset{\sim}{A}_1$ THEN $\underset{\sim}{B}_2$
IF $\underset{\sim}{A}_1$ (THEN $\underset{\sim}{B}_1$) UNLESS $\underset{\sim}{A}_2$

can be decomposed as

IF $\underset{\sim}{A}_1$ THEN $\underset{\sim}{B}_1$
OR
IF $\underset{\sim}{A}_2$ THEN NOT $\underset{\sim}{B}_1$
IF $\underset{\sim}{A}_1$ THEN ($\underset{\sim}{B}_1$) ELSE IF $\underset{\sim}{A}_2$ THEN ($\underset{\sim}{B}_2$)

can be decomposed into the form

IF $\underset{\sim}{A}_1$ THEN $\underset{\sim}{B}_i$
OR
IF NOT $\underset{\sim}{A}_1$ AND IF $\underset{\sim}{A}_2$ THEN $\underset{\sim}{B}_2$

4. **Nested-IF-THEN rules:**

The rule "IF $\underset{\sim}{A}_1$ THEN [IF $\underset{\sim}{A}_2$ THEN ($\underset{\sim}{B}_1$)]" can be of the form

IF $\underset{\sim}{A}_1$ AND $\underset{\sim}{A}_2$ THEN $\underset{\sim}{B}_1$

Thus, based on all the above-mentioned methods compound rules can be decomposed into series ofcanonical simple rules.

AGGREGATION OF RULES

The rule-based system involves more than one rule. Aggregation of rules is the process of obtaining the overall consequents from the individual consequents provided by each rule.

The following two methodsare used foraggregation of fuzzy rules:

1. **Conjunctive system of rules:**

For a system of rules to be jointly satisfied, the rules are connected by "and" connectives. Here, the aggregated output, y, is determined by the fuzzy intersection of all individual ruleconsequents,y_i,where i=1ton, as

$$y = y_1 \text{ and } y_2 \text{ and} \ldots \text{ and } y_n$$

Or

$$y = y_1 \cap y_2 \cap y_3 \cap \cdots \cap y_n$$

This aggregated output can be defined by the membership function

$$\mu_y(y) = \min[\mu_{y_1}(y),\ \mu_{y_2}(y), \ldots, \mu_{y_n}(y)] \quad \text{for} \quad y \in Y$$

2. **Disjunctive system of rules**

In this case, the satisfaction of at least one rule is required. The rules are connected by "or" connectives. Here, the fuzzy union of all individual rule contributions determines the aggregated output, as

$$y = y_1 \text{ or } y_2 \text{ or} \ldots \text{ or } y_n$$

Or

$$y = y_1 \cup y_2 \cup y_3 \cup \cdots \cup y_n$$

Again it can be defined by the membership function

$$\mu_y(y) = \max[\mu_{y_1}(y),\ \mu_{y_2}(y), \ldots \mu_{y_n}(y)] \quad \text{for } y \in Y$$

FUZZY INFERENCE SYSTEMS

Fuzzy rule based systems, fuzzy models, and fuzzy expert systems are generally known as inferencesystems.

The key unit of a fuzzy logic system is FIS. The primary work of this system is decision making.

FIS uses "JF ... THEN" rules along with connectors "OR" or "AND" for making necessary decision rules.

The input to FlS may be fuzzy or crisp, but the output from FIS is always a fuzzy set. When FlSis used asa controller, it is necessary to have crisp output. Hence, there should be a defuzzification unit for convertingfuzzy variables into crisp variables along FIS.

Construction and Working Principle of FIS

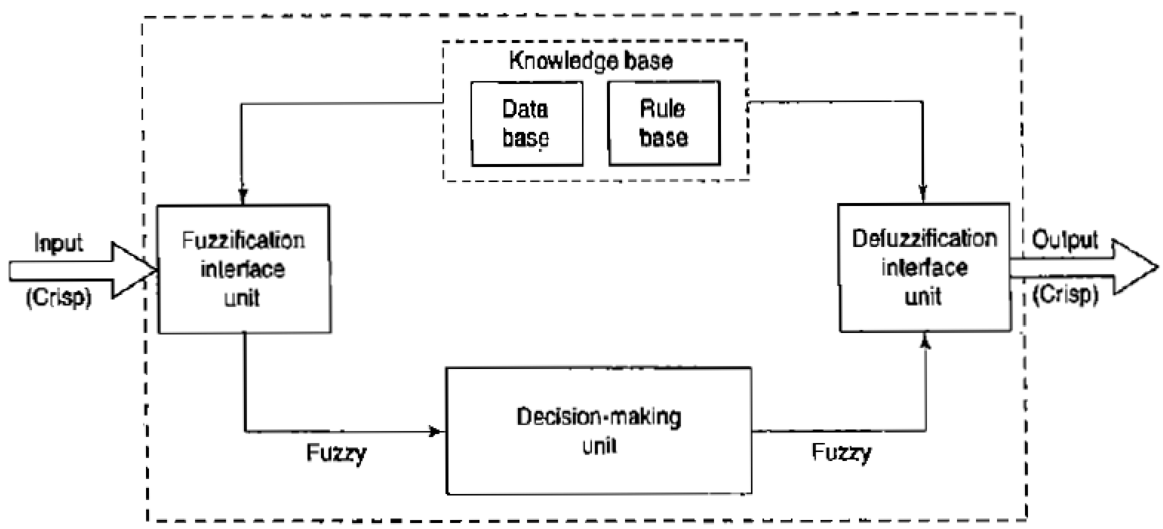
A FIS is constructed of five functional blocks as shown in Figure. They are:

1. A **rule base**that contains numerous fuzzy IF-THEN rules.
2. A **database** that defines the membership functions of fuzzy sets used in fuzzy rules.

3.Decision making unitthat performs operations on the rules.

4.Fuzzification interface unitthat converts the crisp quantities into fuzzy quantities.

5.Defuzzification interface unitthat converts the fuzzy quantities into crisp quantities.



Working methodology of FIS

Initially, in the fuzzif1cation unit, the crisp input is convertedinto a fuzzy input.

Variousfuzzification methods are employed for this.

After this process, rule base is formed.

Database and rule base are collectively called the **knowledge base.**Finally, defuzzification process is carried out to produce crisp output. Mainly, the fuzzy rules are formed in the rule base and suitable decisions are madein the decision-making unit.

Methods of FIS

There are two important types of FIS. They are:
I. Mamdani FIS (1975);
2. Sugeno FIS (1985).

The difference between the two methods lies in the consequent of fuzzy rules. Fuzzy sets are used as ruleconsequents in Mamdani FIS and linear functions of input variables are used as rule consequents inSugeno'smethod. Mamdani’srule finds a greater acceptance in all universal approximators than Sugeno's model.

Mamdani FIS

EbsahimMamdani proposed this system in the year 1975 to control a steam engine and boiler combinationby synthesizing a set of fuzzy rules obtained from people working on the system.

In this case, the output membership functions are expected to be fuzz sets. After aggregation process, each output variable containsa fuzzy set, hence defuzzification is important at the output stage.

The following steps have to be followed to compute the output from this FIS:

Step 1: Determine a *set* of fuzzy rules.
Step 2: Make the inputs fuzzy using input membership functions.

Step 3: Combine the fuzzifiedinputs according to the fuzzy rules for establishing a rule strength.
Step 4: Determine the consequent of the rule by combining the rule strength and the output membershipfunction.
Step 5: Combine all the consequents to get an output distribution.
Step 6: Finally, a defuzzified output distribution is obtained.

The fuzzy rules are formed using "IF-THEN" statements and "AND/OR" connectives.

The consequenceof the rule can be obtained in two steps:

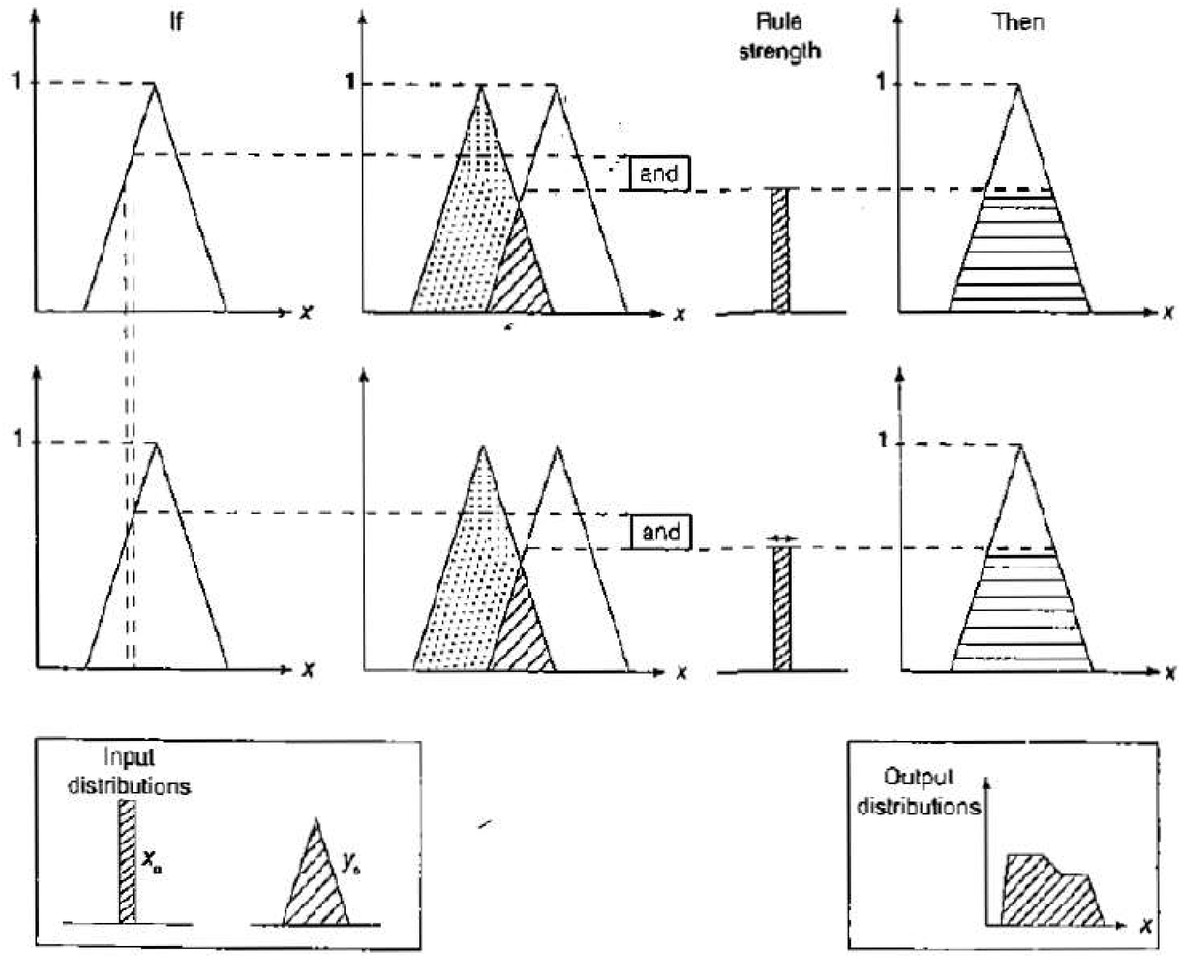
1. By computing the rule strength completely using the fuzzified inputs from the fuzzy combination;
2. By clipping the output membership function at the rule strength.

The outputs of all the fuzzy rules are combined to obtain one fuzzy output distribution. From FIS, it isdesired to get only one crisp output. This crisp output may be obtained from defuzzification process.

Thecommon techniques of defuzzification used are center of mass and Mean of maximum.

Consider a two-input Mamdani FIS with two rules. The model fuzzifies the two inputs by finding theintersection of two crisp input values with the input membership function.

The minimum operation isused to compute the fuzzy input "and" for combining the two fuzzified inputs to obtain a rule strength.The output membership function is clipped at the rule strength. Finally, the maximum operator is used to compute the fuzzy output "or" for combining the output of the two rules. This process is illustrated in the following Figure.



Takagi-Sugeno Fuzzy Model (TS Method)

- Sugeno fuzzy method was proposed by Takagi, Sugeno and Kang in the year 1985. The format of the fuzzyrule of a Sugeno fuzzy model is given by

IF x is A and y is B THEN z=f(x,y)

whereAB are fuzzy sets in the antecedents and z = f (x,y) is a crisp function in the consequent.

- Generally, f (x,y) is a polynomial in the input variables x and y.
- If f(x,y) is a first order polynomial, we get first order Sugeno fuzzy model. If f is a constant, we get zero order Sugeno fuzzy model.
- A zero order Sugeno fuzzy model is functionally equivalent to a radial basis function network under certain minor constraints.

The main steps of the fuzzy inference process namely,

l. fuzzifying the inputs;

2. applying the fuzzy operator

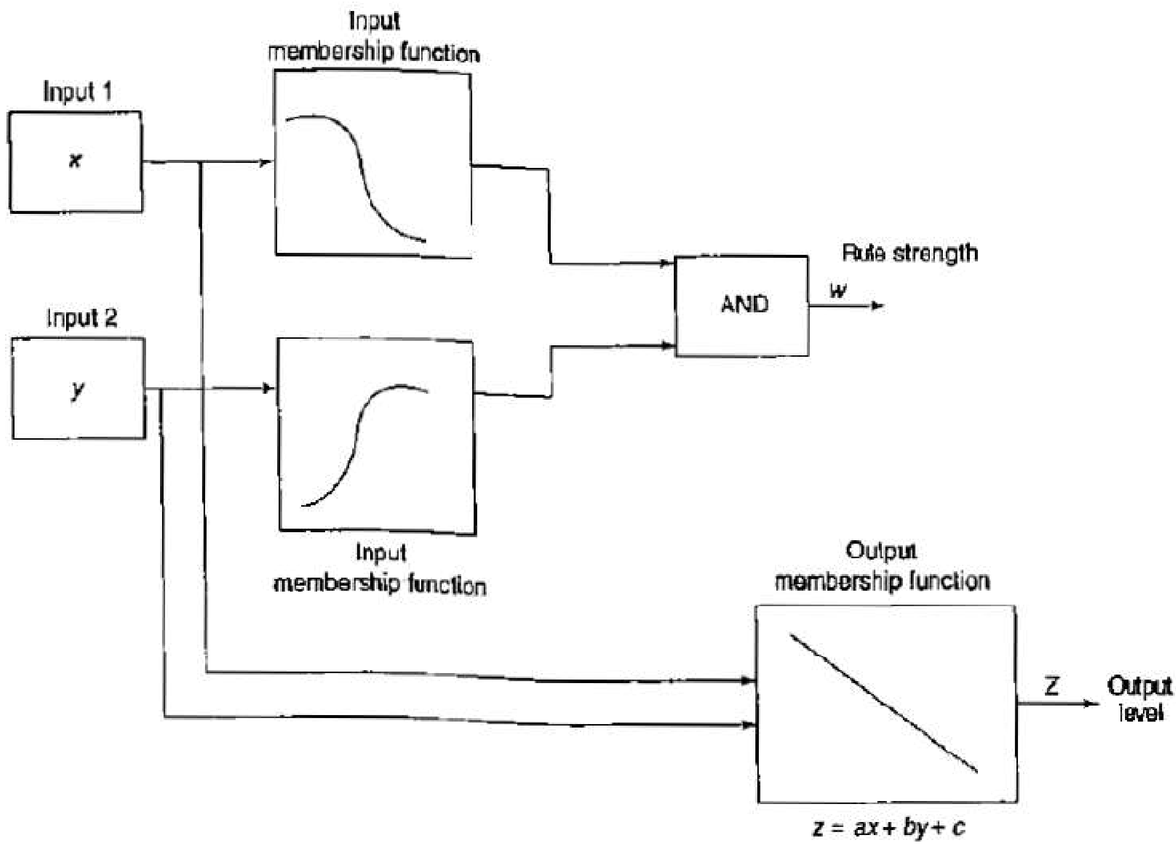
arc exactly the same.

The main difference between Mamdani's and Sugeno's methods is that Sugeno outputmembership functions are either linear or constant.

The rule format of Sugeno form is given by

"If 3=x and 5=y then output is z=ax+by+c."

For a Sugeno model of zero order, the output level z is a constant. The operation of a Sugeno rule is as shownin Figure below:



Comparison between Mamdani and Sugeno Method

The main difference between Mamdani and Sugeno methods lies in the output membership functions. TheSugeno output membership functions are either linear or constant.

The difference also lies in the consequentsof their fuzzy rules and as a result their aggregation and defuzzification procedures differ suitably. A largenumber of fuzzy rules must be employed in Sugeno method for approximating periodic or highly oscillatoryfunctions.

The configuration of Sugeno fuzzy systems can be reduced and it becomes smaller than that ofMamdani fuzzy systems if nontriangular or nontrapezoidal fuzzy input sets are used.

Sugeno controllers havemore adjustable parameters in the rule consequent and the number of parameters grows exponentially with theincrease of the number ofinputvariables. There exist several mathematical results for Sugeno fuzzy controllersthan for Mamdani controllers. Formation of Mamdani ismore easier than Sugeno FIS.

Advantages of Mamdani method

1. It has widespread acceptance;
2. it is well-suitable for human input;
3. it is intuitive.

Advantages of Sugeno method

1. It is computationally efficient.
2. It is compact and works well with linear technique, optimization technique and adaptive technique.
3. It is best suited for mathematical analysis.
4. It has a guaranteed continuity of the output surface

NEURO-FUZZY HYBRID SYSTEMS

Neuro-fuzzy hybrid system (also called fuzzy neural hybrid), proposed by. S. R. Jang, is a learning mechanismthat utilizes the training and learning algorithms from neural networks to find parameters of a fuzzy system(i.e., fuzzy sets, fuzzy rules, fuzzy numbers, and so on).

It can also be defined as a fuzzy system that determinesits parameters by processing data samples by using a learning algorithm derived from or inspired by neuralnetwork theory. Alternately, it is a hybrid intelligent system that fuses artificial neural networks and fuzzylogic by combining the learning and connectionist structure of neural networks with human-like reasoning style of fuzzy systems.

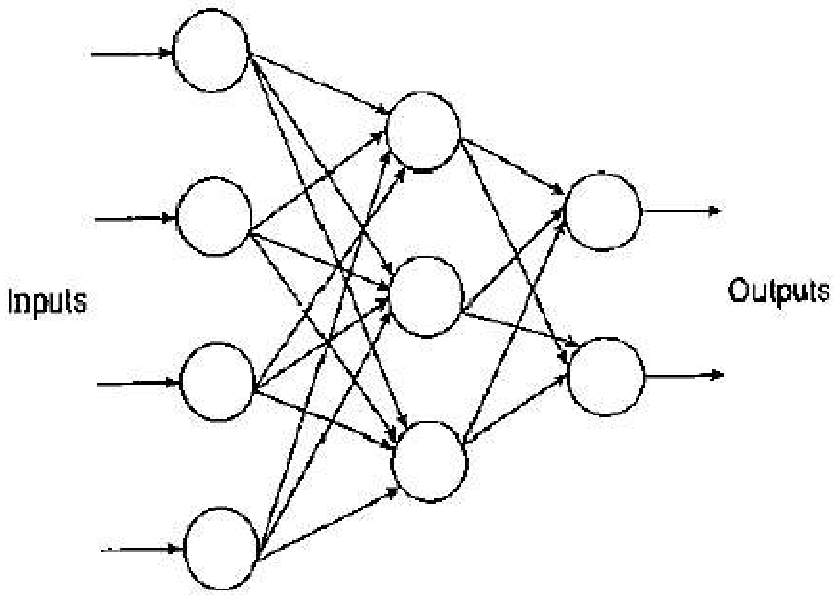
Neuro-fuzzy hybridization is widely termed as Fully Neural Network (FNN) or Neuro-Fuzzy System(NFS).

The neuro-fuzzy is divided into two areas:

l. Linguistic fuzzy modeling focused on interpretability (mainly the Mamdani model).
2. Precise fuzzy modeling focused on accuracy [mainly the Takagi-Sugeno-Kang(TSK) model].

Characteristics of Neuro-Fuzzy Hybrids

The general architecture of neuro-fuzzy hybrid system is as shown in Figure below.



A fuzzy system-based NFSis trained by means of a data-driven learning method derived from neural network theory. This heuristiccauses local changes in the fundamental fuzzy system. At any stage of the learning process- before, during,or after- it can be represented as a set of fuzzy rules. For ensuring the semantic properties of the underlyingfuzzy system, the learning procedure is constrained.
An NFS approximates ann-dimensional unknown function, partly represented by training examples. Thusfuzzy rules can be interpreted as vague prototypes of the training data. *As* shown in Figure above, an NFS isgiven by a three-layer feedforward neural network model.

It can also be observed that the first layer corresponds to the input variables·, and the second and third layers correspond to the fuzzy rules and output variables,respectively.

The fuzzy sets are converted to (fuzzy) connection weights.NFS can also be considered as a system of fuzzy rules wherein the system can be initialized in the formof fuzzy rules based on the prior knowledge available.

Classifications of Neuro-Fuzzy Hybrid Systems

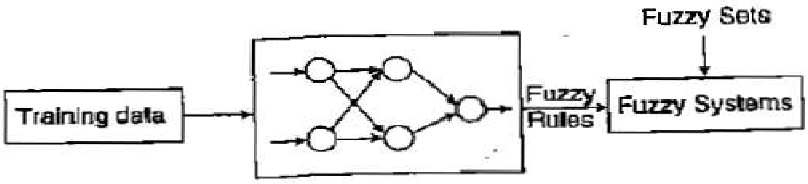
NFSs can be classified into the following two systems:

l. Cooperative NFSs.

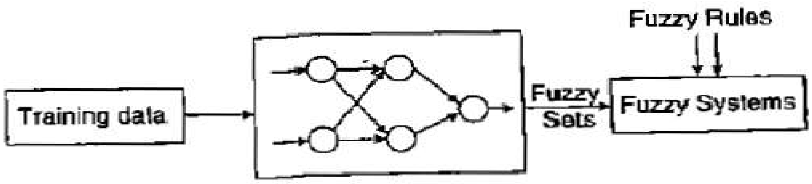
2. General neuro-fuzzy hybrid systems.

Cooperative Neural Fuzzy Systems

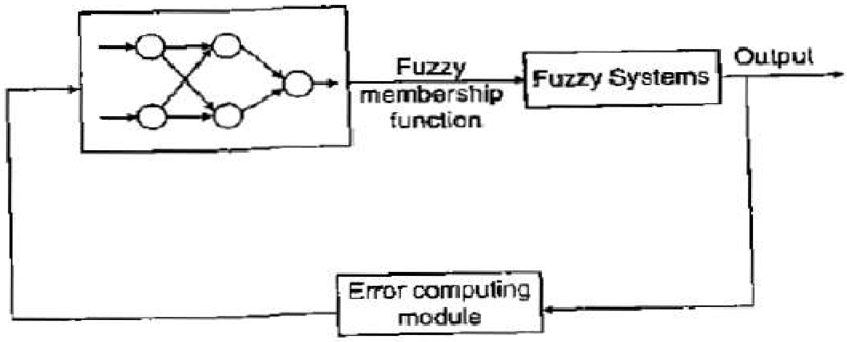
In this type of system, both artificial neural network (ANN) and fuzzy system work independently from eachother. The ANN attempts to learn the parameters from the fuzzy system. Four different kinds of cooperativefuzzy neural networks are shown in Figure below.



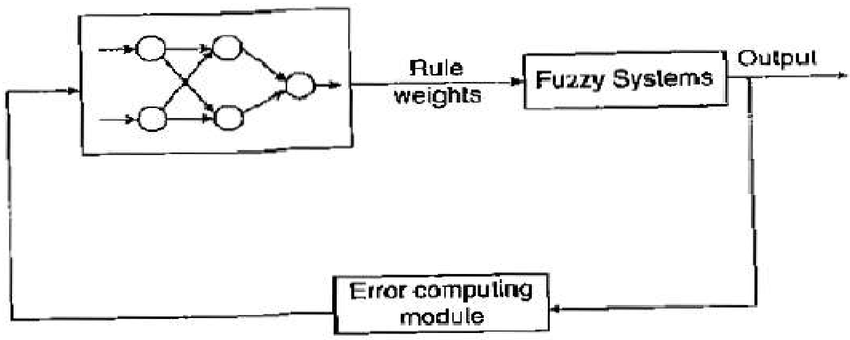
(A)



(B)



(C)



(D)

The FNN in Figure (A) learns fuzzy set from the given training data. This is done, usually, by fittingmembership functions with a neural network; the fuzzy sets then being determined offline. This is followed by their utilization to form the fuzzy system by fuzzy rules that are given, and not learned.

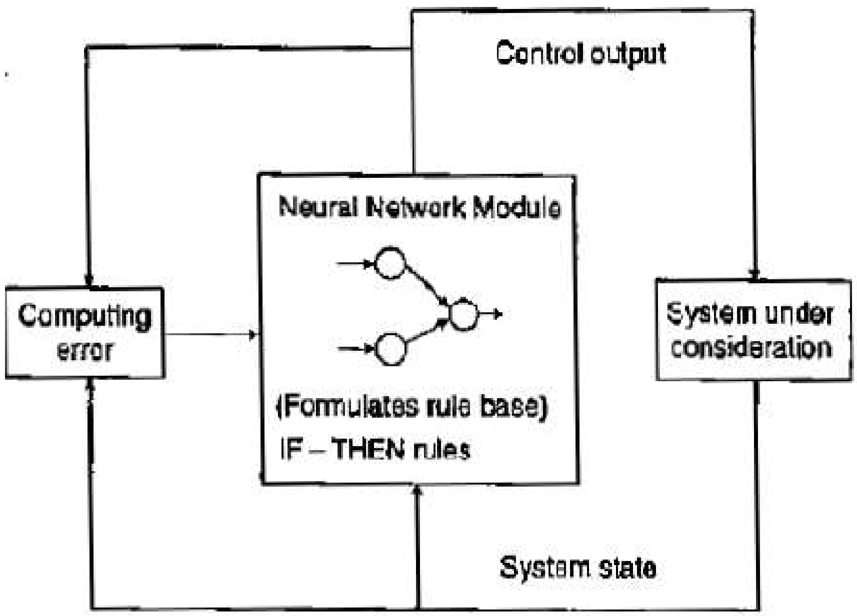
The NFS inFigure (B) determines, by a neural network, the fuzzy rules from the training data. Here again, the neuralnetworks learn offline before the fuzzy system is initialized. The rule learning happens usually by clusteringon self-organizing feature maps. There is also the possibility of applying fuzzy clustering methods to obtainrules.

For the neuro-fuzzy model shown in Figure (C), the parameters of membership function are learntonline, while the fuzzy system is applied. This means that, initially, fuzzy rules and membership functions must be defined beforehand. Also, in order to improve and guide the learning step, the error has to be measured.

The model shown in Figure (D) determines the rule weights for all fuzzy rules by a neural network. Arule is determined by its rule weight-interpreted asthe influence of a rule. They are then multiplied with therule output.

2**) General Neuro-Fuzzy Hybrid Systems (General NFHS)**

General neuro-fuzzy hybrid systems (NFHS) resemble neural networks where a fuzzy system is interpreted as a neural network of special kind. The architecture of general NFHS gives it an advantage because there is nocommunication between fuzzy system and neural network. Figure below illustrates an NFHS.



In this figure therule base of a fuzzy system is assumed to be a neural network; the fuzzy sets are regarded as weights and therules and the input and output variables as neurons. The choice m include or discard neurons can be madein the learning step. Also, the fuzzy knowledge base is represented by the neurons of the neural network; thisovercomes the major drawbacks of both underlying systems.

Membership functions expressing the linguistic terms of the inference rules should be formulated forbuilding a fuzzy controller. However, in fuzzy systems, no formal approach exists to define these functions.

Any shape, such as Gaussian or triangular or bell shaped or trapezoidal, can be considered as a membership function with an arbitrary set of parameters. Thus for fuzzy systems, the optimization of these functionsin terms of generalizing the data is very important; this problem can be solved by using neural networks.

Using learning rules, the neural network must optimize cite parameters by fixing a distinct shape of themembership functions; for example, triangular. But regardless of the shape of the membership functions,training data should also be available.

The neuro fuzzy hybrid systems can also be modeled in an another method. In this case, the trainingdata is grouped into several clusters and each duster is designed to represent a particular rule. These rules aredefined by the crisp data points and are not defined linguistically.

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